Comment on

"Coordinating towards a common good"

by Jorge M. Pacheco and Francisco C. Santos

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Outline

- My understanding of the N–persons Coordination game
- Evolutionary Game Theory
- The prisoner's dilemma
- How can we obtain cooperation from a model?

The N–persons Coordination game

- N players
- strategy of player i: $\sigma_i \in \{C, D\}$
- any outcome determines k contributors
- payoffs:
  - $\pi_D = \alpha b + (1-\alpha) b \cdot \mathbb{I}_{k \geq M}$
  - $\pi_C = \pi_D - c$
- two pure strategies (classes of) Nash Equilibria:
  1. M players contribute and $N - M$ don't
  2. if $M > 1$: no player contributes

A symmetric Nash equilibrium in mixed strategies (1/2)

- Suppose that each agent contributes with probability x
- then she must be indifferent between C and D

  - $$\pi_D = \underbrace{\alpha b + (1-\alpha) b \cdot \sum_{k=M}^{N-1} \binom{N-1}{k} x^k (1-x)^{N-1-k}}_{\equiv f_D(x)}$$
  - $$\pi_C = \underbrace{\alpha b - c + (1-\alpha) b \cdot \sum_{k=M-1}^{N-1} \binom{N-1}{k} x^k (1-x)^{N-1-k}}_{\equiv f_C(x)}$$

A symmetric Nash equilibrium in mixed strategies (2/2)

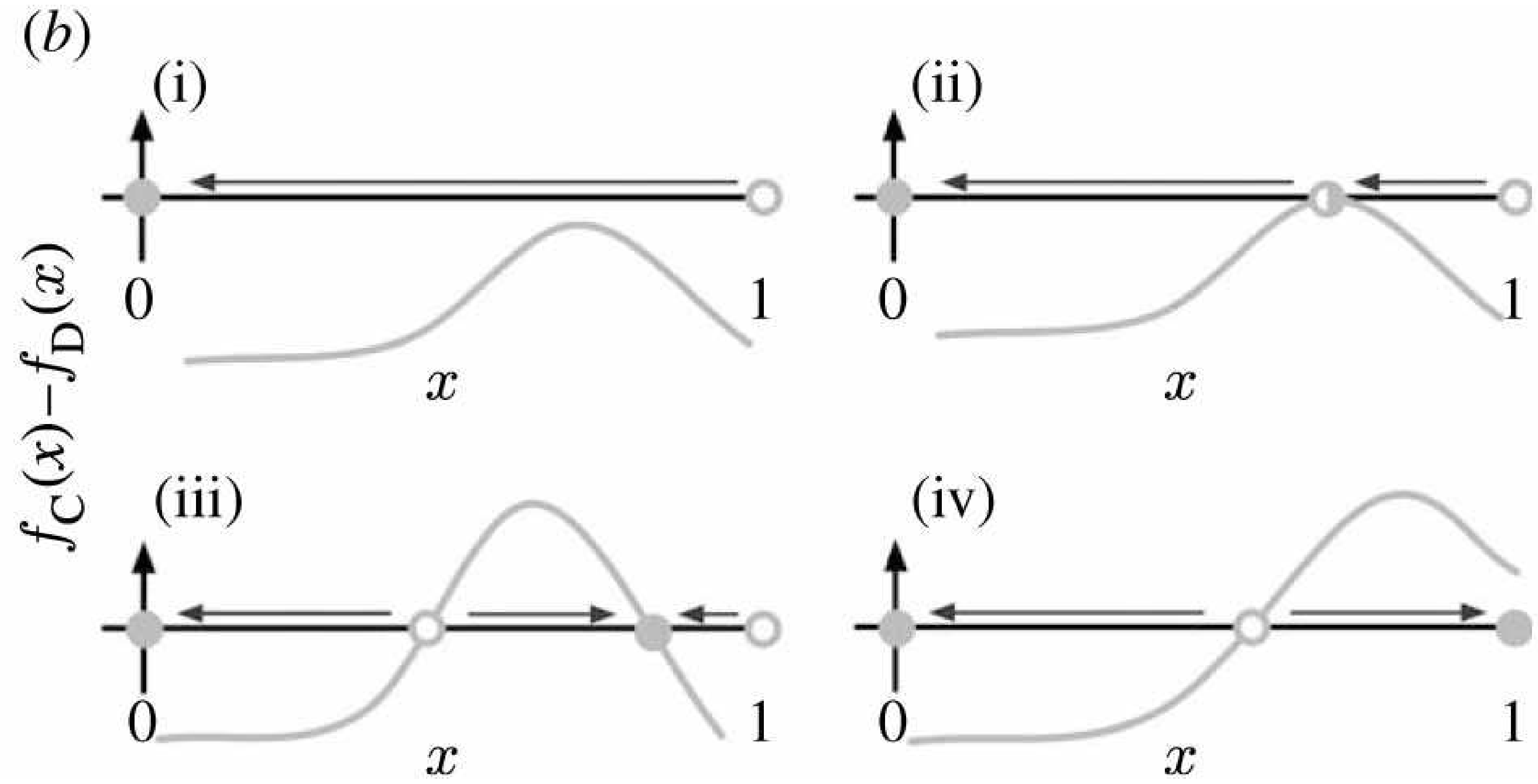


Figure 1 from *Pacheco, Santos, Souza & Skyrms, "Evolutionary dynamics of collective action in N–person stag hunt dilemmas", Proc. Royal Society B (2008)*.

Evolutionary Game Theory

A stochastic process with:

- fitness (payoffs)
- a selection mechanism
- but also mutation (i.e. experimentation)

It is dynamic while Game Theory is only static.

An important result is (van Damme, 1991):

every evolutionarily stable strategy–profile is a Nash equilibrium.

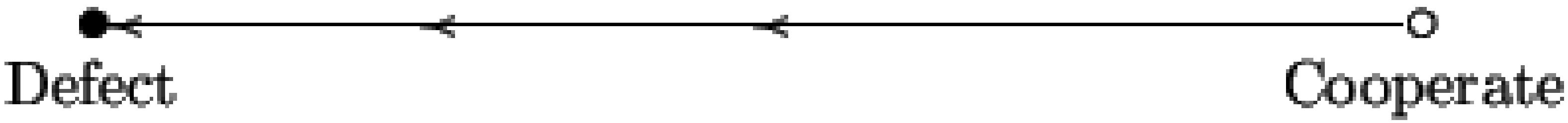
Evolutionary Game Theory helps refining Nash equilibria
($\simeq$ stochastic stability)

The prisoner's dilemma

	C		D	
C	$b,$	b	$0,$	a
D	$a,$	0	$c,$	c

,

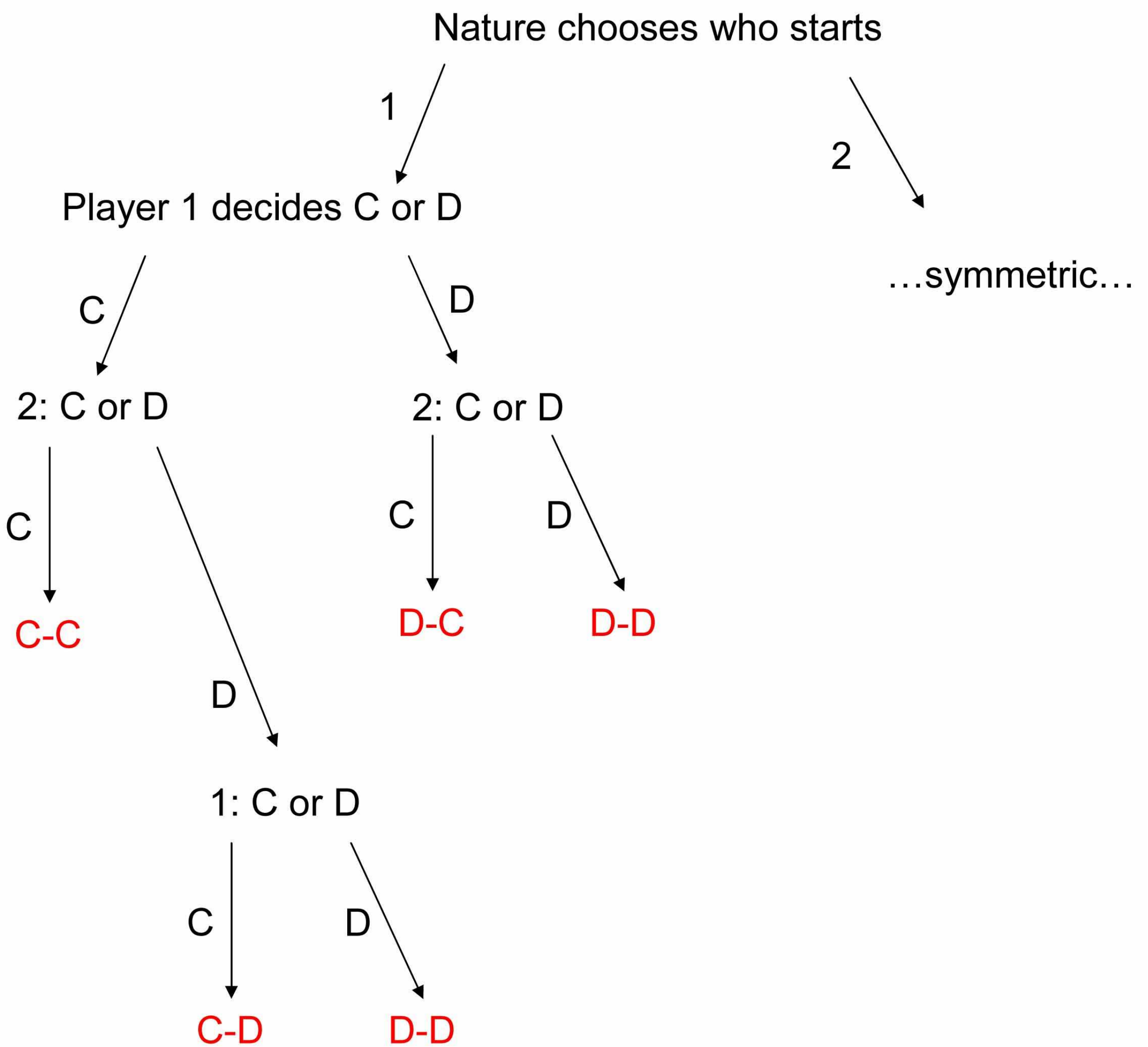
$$a > b > c > 0$$



How can we obtain cooperation (with a different game)?

- Different payoffs (e.g. altruistic utilities)
- Infinitely repeated games (. . . *Folk theorem* . . . which δ? . . .)

An alternative approach (1/2)



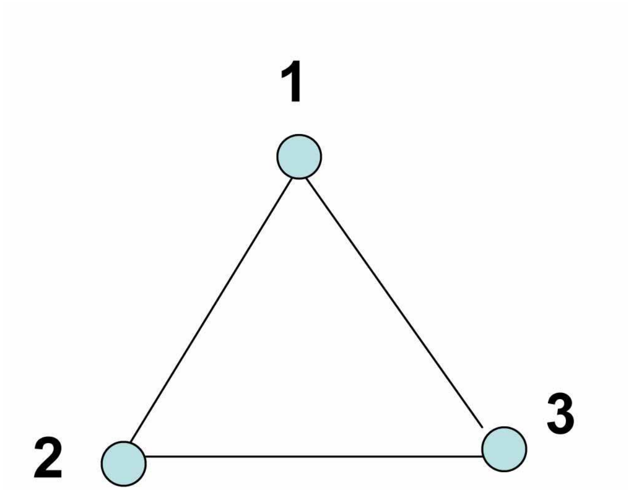
An alternative approach (2/2)

S_t is the *set of contributors* at time t, $n_t = |S_t|$.

1. Nature chooses can ordering on the elements of S_t
   s_t^1 is the first mover, s_t^2 the second mover...
   $i \leftarrow 1$.
   (a) Player s_t^i decides wether she agrees to contribute.
   (b) If she agrees and $i < n_t$, we assign $i \leftarrow i + 1$ and we go back to point (1a).
   (c) If she does not agree and $n_t > 1$, we assign $S_{t+1} \leftarrow S_t \backslash \{s_n^i\}$ and $t \leftarrow t + 1$, and we go back to point (1).
   (d) If she does not agree and $n_t = 1$ payoffs are paid
   (e) If she agrees and $i = n_t$ payoffs are paid

More players

Start with 3 players



Two cases:

1. $\underline{b \leq 2c}$: all contribute ($2b$)
2. $\underline{b \geq 2c}$: first mover free–rides ($2a$) and remaining players cooperate
   (b: non–credible punishment)

Start with N players:

1. a fraction $\gtrsim c/b$ will cooperate
2. all will cooperate only if $c/b \geq \frac{N-2}{N-1}$