Environmental Problems in China: From Kyoto Protocol and Bali Roadmap to Foundations for Cooperative Game-theoretic Solutions

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Abstract

After decades of rapid technological advancement and economic growth, alarming levels of pollutions and environmental degradation are emerging globally. Due to the geographical diffusion of pollutants, unilateral response of one nation or region is often ineffective. Reports are portraying the situation as an industrial civilization on the verge of suicide, destroying its environmental conditions of existence with people being held as prisoners on a runaway catastrophe-bound train. In the last 40 years China has transformed itself from a rural economy to an industrial giant with a significant presence in the world economy. Unfortunately, China's rapid economic development has exerted a significant toll on its natural environment. Though global cooperation in environmental control holds out the best promise of effective action, limited success has been observed. In this paper, we provide an overview of China's environmental problems, examine the impacts of the Kyoto Protocol and Bali Roadmap on China and develop the foundations for a cooperative game-theoretic solution.

Key words: Environmental problem, cooperative game, dynamic stability.

1. Introduction

After decades of rapid technological advancement and economic growth, alarming levels of pollutions and environmental degradation are emerging globally. Due to the geographical diffusion of pollutants, unilateral response of one nation or region is often ineffective. Reports are portraying the situation as an industrial civilization on the verge of suicide, destroying its environmental conditions of existence with people being held as prisoners on a runaway catastrophe-bound train. Though global cooperation in environmental control holds out the best promise of effective action, limited success has been observed. This is the result of many hurdles, ranging from commitment and sharing of costs to disparities in future developments. It is hard to be convinced that multinational joint initiatives like the Kyoto Protocol or pollution permit trading can offer a long-term solution because (i) the plans are limited to a confined set of controls like gas emissions and permits which is unlikely be able to offer an effective mean to reverse the accelerating trend of environmental deterioration, and (ii) there is no guarantee that participants will always be better off and hence be committed within the entire duration of the agreement. In the last 40 years China has transformed itself from a rural economy to an industrial giant with a significant presence in the world economy. This rapid transformation has fueled economic growth that outpaced many of developing countries. China is an active participant in the climate change talks and other multilateral environmental negotiations.

In this paper, we provide an overview of China's environmental problems, examine the impacts of the Kyoto Protocol and Bali Roadmap on China and develop the foundations for a cooperative game-theoretic solution. The paper is organized as follows. Section 2 provides an overview of China's environmental problems. Section

3 considers the impacts of Kyoto Protocol and Bali Roadmap on China's environmental policy. Section 4 presents an analytical model for environmental cooperation in China. Section 5 scrutinizes theoretical principles for successful cooperation. Dynamically stable payment distribution scheme are presented in Section 6 and concluding remarks are given in Section 7.

2. Environmental Problems in China

This rapid economic transformation of China has fueled economic growth that outpaced many of developing countries. Unfortunately, China's rapid economic development has exerted a significant toll on its natural environment. Serious negative consequences of China's rapid industrial development include increased pollution and demise of natural resource base, contaminated water resources, destruction of the natural forests and wetland. Most solid wastes are not properly handled. Pollution becomes a major source of health problems across the country and it causes more than three quarters of a million premature deaths each year. China's polluted environment is largely a result of the country's rapid development and consequently a large increase in primary energy consumption, which is primarily provided by coal power plants.

2.1. Land, Water and Air Pollution

Currently approximately 30% of China's surface area is desert. China's rapid industrialization is causing this area to increase continually. The Gobi desert in the north currently expands by about 950 square miles per year. In the past 50 years, industrial exploitation in the form of dams and other irrigation infrastructure have all but halted the river's natural course, threatening to dry up the entire river valley and convert the plains into a giant dustbowl of unimaginable scale. Recent droughts, deforestation and global warming only serve to bring the region closer to catastrophe.

Inadequate investments in basic water supply and treatment infrastructure have resulted in widespread water pollution. Almost all of the nation's rivers are considered polluted to certain degrees, and half of the population lacks access to clean drinking water. 90% of surface and underground water in urban areas are severely polluted. China classifies its water quality into five grades, from Grade I to Grade V, with Grade V being the most highly polluted[1]. Over 50% of the sections of five of the seven major rivers in China have water quality below Grade IV.

Scarcity of usable water is also a major issue. More than half of China's cities are experiencing severe water supply shortages. Severe water scarcity in Northern China is a serious threat to sustained economic growth and has forced the government to begin implementing a large scale diversion of water from the Yangtze River to northern cities, including Beijing and Tianjin. Official government statistics also

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[1] China Environmental Quality Standards for Surface Water (GB 3838-88) gives out surface water quality standards. Each class of surface water prescribes the quality of the surface water that is necessary for the designated uses.The water bodies are divided into five classes according to the utilization purposes and protection objectives:

Class I is mainly applicable to the water from sources, and the national nature reserves.

Class II is mainly applicable to first class of protected areas for centralized sources of drinking water, the protected areas for rare fishes, and the spawning fields of fishes and shrimps.

Class III is mainly applicable to second class of protected areas for centralized sources of drinking water, protected areas for the common fishes and swimming areas.

Class IV is mainly applicable to the water areas for industrial use and entertainment which is not directly touched by human bodies.

Class V is mainly applicable to the water bodies for agricultural use and landscape requirement.

The water bodies with various functions are classified based on the highest function, and those with seasonal functions may be classified by seasons.

recorded a steady increase in the total volume of wastewater produced: Growing from 29 billion tons in 1981 to 37 billion tons in 1995 to 53.7 billion tons in 2006.

Concerning air pollution, two-thirds of the 338 cities for which air-quality data are available are considered polluted. Respiratory and heart diseases related to air pollution are the leading cause of death in China. Acid rain has been found in one third of the country. China's environmental laws are among the strictest in the world, but enforcing these laws has been difficult in China. The World Health Organization has found that about 750,000 people die prematurely each year from respiratory problems in China.

The total cost of air and water pollution in China in 2003 was 362 billion RMB, or about 2.68% of its GDP (The World Bank and China State Environmental Protection Administration, 2007).

2.2. Changes in Ecosystem

At the end of the 1970s, Chinese economic reform was highly concentrated on enhancing the competititiveness of its economic system. Coastal cities were heavily involved with the reform. Guo (1986) reported that the coastal zone represents 5% of the total area of China or approximately 4.7×10^5 square kilometres. By 1990, coastal areas in China accounted for 55% of its GDP (see Wang, 1992). An annual increase in GDP of the coastal areas by around 10% over the last decade was reported (Cao and Wong, 2007). Pollution is one of the major challenges to sustainability of the coastal areas. Heavy pollution has been found very common in the Chinese river estuaries, bays and coastal areas (Zhou and Zhu, 2006). The sources of pollution come from both land and water. Water based pollutants comes from coastal waters by mariculture and other marine activities and production. Land-based pollutants mainly include riverine exports of agricultural chemicals from coastal catchments, domestic wastes and industrial wastes (Cao and Wong, 2007). Another negative consequence of economic expansion in coastal areas is extensive land reclamation. Until 2002, reclamation has resulted in a conversion of 12 million km^2 from coastal wetland to other land uses since 1949. The coastal wetland losses are about 55% of total coastal wetland in China (Sun, 2004).

Wetland ecosystem has degraded evidently in China. The eco-environment function of the wetland has intensely declined owing to the large scale of agricultural cultivating in Northeast China. The wetland area shrank from 5340×10^2 km^2 in 1949 to 947×10^2 km^2 in 2000 in Sanjiang Plain of northeastern China. In the past two decades, the wetland of the middle and lower range of the Huolin River has declined by 44%. The desertification of the swamp in the Zoige Plateau have also become an important problem as a result of precipitation decline, draining and overgrazing in the past 30 years, with 409.1 km^2 of desertified area and 1202.3 km^2 of potentially desertified area. (Qin et al., 2006).

3. The Impacts of Kyoto Protocol and Bali Roadmap on China's Environmental Policy

China is an active participant in the climate change talks and other multilateral environmental negotiations. It is a signatory to the Basel Convention governing the transport and disposal of hazardous waste and the Montreal Protocol for the Protection of the Ozone Layer, as well as the Convention on International Trade in Endangered Species and the Kyoto Protocol, although China is not required to reduce its carbon emissions under the terms of the present agreement. On June 19, 2007, the

Netherlands Environmental Assessment Agency announced, based on an analysis of fossil fuel consumption (including specially the coal power plants) and cement production data, that China surpassed the United States as the world's largest emitter of carbon dioxide, putting out 6,200 million tons, to America's 5,800 million (Netherlands Environmental Assessment Agency, 2007).

3.1. Kyoto Protocol

International cooperation and the provision of support to developing countries and to countries with economies in transition are crucial to ensure the implementation of the Kyoto Protocol by all Parties. The Convention divides countries into three main groups according to differing commitments: Annex I Parties; Annex II Parties[2]; and Non-Annex I Parties. The 48 Parties, classified as least developed countries (LDCs) by the United Nations, are given special consideration under the Convention on account of their limited capacity to respond to climate change and adapt to its adverse effects. Parties are urged to take full account of the special situation of LDCs when considering funding and technology-transfer activities.

China signed the pact on 29 May 1998. And then on 3 September 2002 at the World Summit on Sustainable Development, China approved the Kyoto Protocol to the United Nations Framework Convention on Climate Change. As one of the non-Annex I Parties to the Convention, China insists that the gas emissions level of any given country should the computed as the product of its per capita emission and its population. China considers the criticism of its energy policy unjust. However,

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[2] Annex II Parties consist of the OECD members of Annex I, but not the EIT Parties. They are required to provide financial resources to enable developing countries to undertake emissions reduction activities under the Convention and to help them adapt to adverse effects of climate change. In addition, they have to "take all practicable steps" to promote the development and transfer of environmentally friendly technologies to EIT Parties and developing countries. Funding provided by Annex II Parties is channelled mostly through the Convention's financial mechanism.

China is currently the second largest emitter of GHGs, while NewScientist (2006) predicted that China will surpass the USA in GHG emission USA in 2 or 3 years.

After “Kyoto Protocol” took effect in 2005 China, as a developing country that is vulnerable to the climate change, has to deal with GHG emissions reduction, economic growth, resource scarcity, population control and clean technology development simultaneously.

3.2. The Bali Roadmap: Towards a Binding Framework

The 15-day U.N. climate change conference in Bali ended on 15 Dec 2007 with the adoption of a Bali roadmap, which is expected to launch negotiations on a crucial international climate change regime in Copenhagen in December 2009. Over 180 countries (including the United States and Australia) agreed to a clear agenda for the key issues to be negotiated up to 2009 for a new treaty, including actions for adapting to the negative consequences of climate change, ways to reduce GHG emissions, ways to deploy climate-friendly technologies and financing both adaptation and mitigation measures.

The climate conference adopted a plan to negotiate a new global warming pact, after the United States suddenly reversed its opposition to a call by developing nations for technological help to battle rising temperatures. Upcoming talks, to be completed in 2009, may help determine for years to come how well the world can control climate change, and how severe the consequences of global warming will be. Developed countries agreed to “measurable, reportable and verifiable nationally appropriate mitigation commitments or actions, including quantified emission limitation and reduction objectives” as well. For developing countries, they made compromise to accept the “measurable, reportable and verifiable nationally appropriate mitigation actions in the context of sustainable development.” The Bali roadmap suggested an ambitious goal for cutting the emissions by industrial nations -- by 25 to 40 percent below the 1990 levels by 2020.

In the years preceding to the Cophenhagen Conference China has to prepare itself for new obligatons that might appear. Once again clean technology development, resource use and economic competitiveness are the issues that China is concerned with.

4. An Analytical Model for Environmental Cooperation in China

Though cooperation in environmental control holds out the best promise of effective action, limited success has been observed. Existing multinational joint initiatives like the Kyoto Protocol, Bali Roadmap and the upcoming Copenhagen Conference can hardly be expected to offer a long-term solution because (i) the plans are limited to a confined set of controls like gas emissions and permits which is unlikely be able to offer an effective mean to reverse the accelerating trend of environmental deterioration in China, and (ii) there is no guarantee that China as a participants will always be better off and hence be committed within the entire duration of the agreement.

A cooperative solution involves a set of environmental policy instruments. The implementation of such a scheme would inevitably bring about different implications in cost and benefit to each of the participating nations. To construct a cooperative solution that China would commit to from beginning to end, the proposed arrangement must guarantee that every participant will be better-off and the originally agreed upon arrangement remain effective at any time within the cooperative period for any feasible state brought about by prior optimal behavior. Hence a cooperative solution satisfying this requirement has to be sought.

In this section we present an analytical framework for environmental cooperation in China.

4.1. The Industrial Sector

Consider a n nations international community including China. At time instant s the demand system of the outputs of the nations is:

$$P_i(s) = f^i[q_1(s), q_2(s), \cdots, q_n(s), s], \qquad i \in N \equiv \{1, 2, \cdots, n\}, \tag{4.1}$$

where $P_i(s)$ is the price vector of the output vector of nation i and $q_j(s)$ is the output of nation j. Industrial profits of nation i at time s can be expressed as:

$$f^i[q_1(s), q_2(s), \cdots, q_n(s), s]q_i(s) - c^i[q_i(s), v_i(s)], \quad \text{for } i \in N, \tag{4.2}$$

where $v_i(s)$ is the set of environmental policy instruments of government i and $c^i[q_i(s), v_i(s)]$ is the cost of producing $q_i(s)$ under policy $v_i(s)$. The term $v_i(s)$ represents nation i's comprehensive set of policy instruments including taxes, subsidies, technology choices, pollution abatement activities, pollution legislations and green technology R&D. Profit maximization by the industrial sectors yields:

$$f^i[q_1(s), q_2(s), \cdots, q_n(s), s] + f^i_{q_i}[q_1(s), q_2(s), \cdots, q_n(s), s]q_i(s) - c^i_{q_i}[q_i(s), v_i(s)] = 0,$$

$$\text{for } i \in N. \tag{4.3}$$

Condition (4.3) is a system of implicit functions in $q(s) = [q_1(s), q_2(s), \cdots, q_n(s)]$

with government policies $v(s)=[v_1(s),v_2(s),\cdots,v_n(s)]$ being regarded as parameters. The existence of a market equilibrium reflects the satisfaction of the Implicit Function Theorem in (4.3) and nation i's instantaneous market equilibrium output can be expressed as:

$$q_i^*(s)=\hat{q}^i[v_1(s),v_2(s),\cdots,v_n(s),s]\equiv\hat{q}^i[v(s),s], \qquad \text{for} \quad i\in N. \tag{4.4}$$

One can readily observe from (4.4) that each nation's output decision depends on government environmental policies.

4.2. Accumulation Dynamics of Pollutants

Industrial production emits pollutants into the environment and the amount of pollution created by different nations' outputs may be different. The pollutant will then add to the stock of existing pollution. Each government adopts its own pollution abatement policy to reduce the pollution stock. Let $x(s)\subset R^m$ denote the level of pollution at time s, the dynamics of pollution stock is governed by the stochastic differential equation:

$$dx(s)=\left[\sum_{j=1}^{n}a_j[q_j(s),v_j(s)]-\sum_{j=1}^{n}b_j[u_j(s),x(s)]-\delta[x(s)]x(s)\right]ds+\sigma[x(s)]dz(s),$$

$$x(t_0)=x_{t_0}, \tag{4.5}$$

where σ is a noise parameter and $z(s)$ is a Wiener process,

$a_j[q_j(s),v_j(s)]$ is the amount of pollution created by $q_j(s)$ amount of output produced under policy $v_i(s)$,

$u_j(s)$ is the pollution abatement effort of nation j,

$b_j[u_j(s), x(s)]$ is the amount of pollution removed by $u_j(s)$ unit of abatement effort of nation j, and $\delta[x(s)]$ is the natural rate of decay of the pollutants. Moreover, $\delta(x)$ is negatively related to x reflecting the phenomenon that the natural rate of decay declines as the level of pollution stock rises. The stochastic nature of (4.5) reflects the uncertainty in the evolution of the pollution stock.

4.3. The Governments' Objectives

The governments have to promote business interests and at the same time handle the financing of the costs brought about by pollution. In particular, each government maximizes the gains in the industrial sector plus tax revenue minus expenditures on pollution abatement and damages from pollution. The instantaneous objective of government i at time s can be expressed as:

$$f^i[q_1(s), q_2(s), \cdots, q_n(s), s]q_i(s) - c^i[q_i(s), v_i(s)] - c_i^P[v_i(s)] - c_i^a[u_i(s)] - h_i[x(s)],$$
$$i \in N, \tag{4.6}$$

where $c_i^P[v_i(s)]$ is the cost of implementing the vector policy instrument $v_i(s)$, $c_i^a[u_i(s)]$ is the cost of employing u_i amount of pollution abatement effort, and $h_i[x(s)]$ is the value of damage to country i from $x(s)$ amount of pollution.

The governments' planning horizon is $[t_0, T]$. It is possible that T may be very large. The discount rate is r. At time T, the terminal appraisal of pollution damage is $g^i[x(T)]$ where $\partial g^i / \partial x < 0$. Each one of the n governments seeks to maximize the integral of its instantaneous objective (4.6) over the planning horizon subject to pollution dynamics (4.5) with controls on the level of abatement effort and output tax.

Substitute $q_i(s)$, for $i \in N$, from (4.4) into (4.5) and (4.6) one obtains a stochastic differential game in which government $i \in N$ seeks to:

$$\max_{v_i(s),u_i(s)} E_{t_0} \left\{ \int_{t_0}^{T} \Bigg[f^i\{\hat{q}^1[v(s),s], \hat{q}^2[v(s),s], \cdots, \hat{q}^n[v(s),s], s\}\hat{q}^i[v(s),s] \right.$$

$$- c^i\{\hat{q}^i[v(s),s], v_i(s)\} - c_i^P[v_i(s)] - c_i^a[u_i(s)] - h_i[x(s)] \Bigg] e^{-r(s-t_0)} ds$$

$$\left. + g^i[x(T)]e^{-r(T-t_0)} \right\} \tag{4.7}$$

subject to

$$dx(s) = \left[\sum_{j=1}^{n} a_j\{\hat{q}^j[v(s),s], v_j(s)\} - \sum_{j=1}^{n} b_j[u_j(s), x(s)] - \delta[x(s)]x(s) \right] ds + \sigma[x(s)]dz(s),$$

$$x(t_0) = x_{t_0}, \tag{4.8}$$

4.4. Noncooperative Outcomes Characterization

Since the payoffs of nations are measured in monetary terms, the game (4.7)-(4.8) is a transferable payoff game. Under a noncooperative framework, a feedback Nash equilibrium solution (if it exists) can be characterized as (see Basar and Olsder, 1995, and Yeung and Petrosyan, 2006):

Definition 4.1. A set of feedback strategies $\{u_i^*(t) = \mu_i(t,x), v_i^*(t) = \phi_i(t,x)$, for $i \in N\}$ provides a Nash equilibrium solution to the game (4.7)-(4.8) if there exist suitably smooth functions $V^{(t_0)i}(t,x) : [t_0,T] \times R \to R$, $i \in N$, satisfying the following partial differential equations:

$$-V_t^{(t_0)i}(t,x) - \frac{\sigma^2 x^2}{2} V_{xx}^i(t,x) - \frac{1}{2}\sum_{k,j} \sigma^{kj}(x) V_{x_k x_j}^{(t_0)i}(t,x)$$

$$\begin{aligned}
&= \max_{v_i, u_i} \Bigg\{ \Bigg[\quad f^i\{\hat{q}^1[v_i, \phi_{\neq i}(t,x), t], \hat{q}^2[v_i, \phi_{\neq i}(t,x), t], \cdots \\
&\cdots, \hat{q}^n[v_i, \phi_{\neq i}(t,x), t)]\}\hat{q}^i[v_i, \phi_{\neq i}(t,x), t] - c\{\hat{q}^i[v_i, \phi_{\neq i}(t,x), t], v_i\} - c_i^P[v_i] - c_i^a[u_i] \\
&- h_i(x) \quad \Bigg] e^{-r(t-t_0)} \\
&+ V_x^{(t_0)i} \Bigg[\sum_{j=1}^{n} a_j\{\hat{q}^j[v_i, \phi_{\neq i}(t,x), t], v_j\} - b_i(u_i, x) - \sum_{\substack{j=1 \\ j\neq i}}^{n} b_j[\mu_j(t,x), x] - \delta(x)x \quad \Bigg] \Bigg\}, \qquad (4.9)
\end{aligned}$$

$$V^{(t_0)i}(T,x) = g^i[x]e^{-r(T-t_0)}, \qquad (4.10)$$

where

$$\phi_{\neq i}(t,x) = [\phi^1(t,x), \phi^2(t,x), \cdots, \phi^{i-1}(t,x), \phi^{i+1}(t,x), \cdots, \phi^n(t,x)]. \qquad \blacksquare$$

In a prevailing Nash equilibrium the function $V^{(t_0)i}(t,x)$ is then the integral:

$$\begin{aligned}
E_{t_0} \Bigg\{ \int_t^T \Bigg[\quad & f^i\{\hat{q}^1[\phi(s,x(s)), s], \hat{q}^2[\phi(s,x(s)), s], \cdots \\
& \cdots, \hat{q}^n[\phi(s,x(s)), s], s\}\hat{q}^i[\phi(s,x(s)), s] - c^i\{\hat{q}^i[\phi(s,x(s)), s], \phi_i(s,x(s))\} \\
& - c_i^P[\phi_i(s,x(s))] - c_i^a[\mu_i(s,x(s))] - h_i[x(s)] \quad \Bigg] e^{-r(s-t_0)} ds \\
& + g^i[x(T)]e^{-r(T-t_0)} \, \Bigg| \, x(t) = x \Bigg\} . \qquad \text{for } i \in N . \qquad (4.11)
\end{aligned}$$

The game equilibrium dynamics then becomes:

$$\begin{aligned}
dx(s) = \Bigg[\; & \sum_{j=1}^{n} a_j\{\hat{q}^j[\phi(s,x(s)), s], \phi_j(s,x(s))\} - \sum_{j=1}^{n} b_j[\mu_j(s,x(s)), x(s)] - \delta[x(s)]x(s) \; \Bigg] ds \\
& + \sigma x(s)dz(s), \qquad x(t_0) = x_{t_0} . \qquad (4.12)
\end{aligned}$$

Remark 4.1. One can readily verify that $V^{(\tau)i}(t,x_t)=V^{(t_0)i}(t,x_t)\ e^{r(\tau-t_0)}$, for $\tau\in[t_0,T]$, is the value function to player i at time $t\in[\tau,T]$ when the state $x(t)=x_t$ in the game (4.7) - (4.8) which starts at time τ. ♦

5. Theoretical Principles for Successful Cooperation

Now consider the case when all the nations want to cooperate and agree to act so that an international optimum could be achieved. For the cooperative scheme to be upheld throughout the game horizon both group rationality and individual rationality are required to be satisfied at any time. Group optimality ensures that all potential gains from cooperation are captured. Failure to fulfill group optimality leads to condition where the participants prefer to deviate from the agreed upon solution plan in order to extract the unexploited gains. Individual rationality is required to hold so that the payoff allocated to a nation under cooperation will be no less than its noncooperative payoff. Failure to guarantee individual rationality leads to condition where the concerned participants would reject the agreed upon solution plan and play noncooperatively.

Finally, as mentioned in Section 1, to ensure that the cooperative solution is dynamically consistent, the agreement must be subgame-consistent. In the absence of a punishment scheme, the cooperative plan will dissolve if any of the nations deviates from the agreed-upon plan.

5.1. Group Optimality and Cooperative State Trajectory

Consider the cooperative stochastic differential games with payoff structure (4.7) and dynamics (4.8). To secure group optimality the participating nations seek to maximize their joint expected payoff by solving the following stochastic control problem:

$$\max_{v_1,v_2,\cdots,v_n;u_1,u_2,\cdots,u_n} E_{t_0}\left\{\int_{t_0}^{T}\left[\sum_{i=1}^{n} f^i\{[\hat{q}^1[v(s),s],\hat{q}^2[v(s),s],\cdots,\hat{q}^n[v(s),s],s]\}\hat{q}^i[v(s),s]\right.\right.$$

$$-c^i\{\hat{q}^i[v(s),s],v_i(s)\}-c_i^P[v_i(s)]-c_i^a[u_i(s)]$$

$$\left.\left.-h_i[x(s)]\right]e^{-r(t-t_0)}ds+\sum_{i=1}^{n} g^i[x(T)]e^{-r(T-t_0)}\right\} \tag{5.1}$$

subject to (4.8).

Invoking Fleming's (1969) technique in stochastic control a set of controls $\{[v_i^*(t),u_i^*(t)]=[\psi_i(t,x),\varpi_i(t,x)]$, for $i\in N\}$ constitutes an optimal solution to the stochastic control problem (5.1) and (4.8) if there exists continuously differentiable function $W^{(t_0)}(t,x):[t_0,T]\times R\rightarrow R$, $i\in N$, satisfying the following partial differential equations:

$$-W_t^{(t_0)}(t,x)-\frac{1}{2}\sum_{k,j}\sigma^{kj}(x)W_{x_kx_j}^{(t_0)}(t,x)$$

$$=\max_{v_1,v_2,\cdots,v_n;u_1,u_2,\cdots,u_n}\left\{\sum_{i=1}^{n} f^i[\hat{q}^1(v,t),\hat{q}^2(v,t),\cdots\right.$$

$$\left.\cdots,\hat{q}^n(v,t),t]\hat{q}^i(v,t)-c^i[\hat{q}^i(v,t),v_i]-c_i^P(v_i)-c_i^a(u_i)-h_i(x)\right]e^{-r(t-t_0)}$$

$$\left.+W_x^{(t_0)}(t,x)\left[\sum_{j=1}^{n}a_j[\hat{q}^j(v,t),v_j]-\sum_{j=1}^{n}b_j(u_j,x)-\delta(x)\,x\right]\right\}, \text{ and}$$

$$W^{(t_0)}(T,x)=\sum_{i=1}^{n} g^i(x)e^{-r(T-t_0)}. \tag{5.2}$$

Hence the players will adopt the cooperative control $\{[\psi_i(t,x),\varpi_i(t,x)]$, for

$i \in N$ and $t \in [t_0, T]$ $\}$. The value function $W^{(t_0)}(t,x)$ is then the integral:

$$\begin{aligned}
&E_{t_0}\Bigg\{ \int_t^T \Bigg[\sum_{i=1}^{n} f^i\{\hat{q}^1[\psi(s,x(s)),s],\hat{q}^2[\psi(s,x(s)),s],\cdots \\
&\qquad \cdots,\hat{q}^n[\psi(s,x(s)),s],s\}\hat{q}^i[\psi(s,x(s)),s]-c^i\{\hat{q}^i[\psi(s,x(s)),s],\psi_i(s,x(s))\} \\
&\qquad -c_i^P[\psi_i(s,x(s))]-c_i^a[\varpi_i(s,x(s))]-h_i[x(s)] \Bigg] e^{-r(s-t_0)}ds \\
&\qquad +\sum_{i=1}^{n} g^i[x(T)]e^{-r(T-t_0)} \Bigg| \; x(t)=x \Bigg\}. \qquad \text{for } i \in N. \qquad (5.3)
\end{aligned}$$

The optimal trajectory under cooperation becomes

$$\begin{aligned}
&dx(s)=\Bigg[\sum_{j=1}^{n} a_j\{\hat{q}^j[\psi(s,x(s)),s],\psi_j(s,x(s))\}-\sum_{j=1}^{n} b_j[\varpi_j(s,x(s)),x(s)]-\delta[x(s)]x(s) \Bigg]ds \\
&\qquad +\sigma x(s)dz(s), \qquad x(t_0)=x_{t_0}. \qquad (5.4)
\end{aligned}$$

The solution to (5.4) can be expressed as:

$$\begin{aligned}
&x^*(t)=x_0+ \\
&\int_{t_0}^{t} \Bigg\{ \sum_{j=1}^{n} a_j\{\hat{q}^j[\psi(s,x^*(s)),s],\psi_j(s,x^*(s))\}-\sum_{j=1}^{n} b_j[\varpi_j(s,x^*(s)),x^*(s)]-\delta[x^*(s)]x^*(s) \Bigg\}ds \\
&\qquad +\int_{t_0}^{t} \sigma x^*(s)dz(s). \qquad (5.5)
\end{aligned}$$

We use X_t^* to denote the set of realizable values of $x^*(t)$ at time t generated by (5.5). The term x_t^* is used to denote an element in the set X_t^*.

The cooperative control for the game $\Gamma_c(x_0, T-t_0)$ over the time interval $[t_0, T]$ can be expressed more precisely as:

$$\psi_i\,(t, x^*(t)) \text{ and } \varpi_i\,(t, x^*(t)) \text{ for } t \in [t_0, T] \text{ and } i \in N. \tag{5.6}$$

Note that for group optimality to be achievable, the cooperative controls (5.6) must be exercised throughout time interval $[t_0, T]$.

Remark 5.1. One can readily verify that $W^{(\tau)}(t, x_t^*) = W^{(t_0)}(t, x_t^*)\ e^{r(\tau - t_0)}$, for $\tau \in [t_0, T]$, is the value function at time $t \in [\tau, T]$ of the stochastic control problem (4.8) and (5.1) which starts at time τ with $x(t) = x_t^* \in X_t^*$. ♦

5.2. Individually Rationality

An agreed upon optimality principle must be sought to allocate the cooperative payoff. In a dynamic framework individual rationality has to be maintained at every instant of time within the cooperative duration $[t_0, T]$ given any feasible state generated by the cooperative trajectory (5.9). For $\tau \in [t_0, T]$, let $\xi^{(\tau)i}(\tau, x_\tau^*)$ denote the solution imputation (payoff under cooperation) over the period $[\tau, T]$ to player $i \in N$ given that the state is $x_\tau^* \in X_\tau^*$. Individual rationality along the cooperative trajectory requires:

$$\xi^{(\tau)i}(\tau, x_\tau^*) \geq V^{(\tau)i}(\tau, x_\tau^*), \quad \text{for } i \in N,\ x_\tau^* \in X_\tau^* \text{ and } \tau \in [t_0, T]. \tag{5.7}$$

Since nations are asymmetric and the number of nations may be large, a reasonable solution optimality principle for gain distribution is to share the expected gain from cooperation proportional to the nations' relative sizes of expected noncooperative payoffs.

5.3. Subgame-consistent Imputation

As mentioned before, a very stringent condition -- subgame consistency -- is required for a credible cooperative solution under a dynamic stochastic framework. In particular, the solution optimality principle must be maintained in any subgame which starts at a later time with any feasible state brought about by prior optimal behaviors so that no player has incentives to deviate from the previously adopted optimal behavior throughout the game.

In order to satisfy the property of subgame consistency, the optimality principle of sharing the expected gain proportional to the nations' relative sizes of expected noncooperative payoffs has to remain in effect throughout the cooperation period. Hence the solution imputation scheme $\{\xi^{(\tau)i}(\tau, x_\tau^*)$; for $i \in N\}$ has to satisfy:

Condition 5.1.

$$\begin{aligned}\xi^{(\tau)i}(\tau,x_\tau^*) &= V^{(\tau)i}(\tau,x_\tau^*) + \frac{V^{(\tau)i}(\tau,x_\tau^*)}{\sum_{j=1}^{n} V^{(\tau)j}(\tau,x_\tau^*)} \left[W^{(\tau)}(\tau,x_\tau^*) - \sum_{j=1}^{n} V^{(\tau)j}(\tau,x_\tau^*) \right] \\ &= \frac{V^{(\tau)i}(\tau,x_\tau^*)}{\sum_{j=1}^{n} V^{(\tau)j}(\tau,x_\tau^*)} W^{(\tau)}(\tau,x_\tau^*),\end{aligned} \tag{5.8}$$

for $i \in N$, $x_\tau^* \in X_\tau^*$ and $\tau \in [t_0, T]$. ♦

One can easily verify that the imputation scheme in Condition 5.1 satisfies both group optimality and individual rationality. Crucial to the analysis is the formulation of a payment distribution mechanism that would lead to the realization of Condition 5.1. This will be done in the next Section.

6. Dynamically Stable Payment Distribution

Following Yeung and Petrosyan (2004 and 2006), we formulate a payment distribution scheme over time so that the agreed upon imputation (5.8) can be realized for any time instant $\tau \in [t_0, T]$ with the state being $x_\tau^* \in X_\tau^*$. Let the vectors $B(s, x_s^*)$ = $[B_1(s, x_s^*), B_2(s, x_s^*), \cdots, B_n(s, x_s^*)]$ denote the instantaneous payment to the n nations at time instant s when the state is $x_s^* \in X_s^*$. A terminal value of $g^i[x_T^*]$ is realized by nation i at time T.

To satisfy (5.8) it is required that

$$\xi^{(\tau)i}(\tau, x_\tau^*) = \frac{V^{(\tau)i}(\tau, x_\tau^*)}{\sum_{j=1}^{n} V^{(\tau)j}(\tau, x_\tau^*)} W^{(\tau)}(\tau, x_\tau^*)$$

$$= E_\tau \left\{ \left(\int_\tau^T B_i(s, x^*(s)) e^{-r(s-\tau)} ds + g^i[x_T^*] e^{-r(T-\tau)} \,\middle|\, x(\tau) = x_\tau^* \right. \right\},$$

for $i \in N$, $x_\tau^* \in X_\tau^*$ and $\tau \in [t_0, T]$. (6.1)

To facilitate further exposition, we use the term $\xi^{(\tau)i}(t, x_t^*)$ which equals

$$E_\tau \left\{ \left(\int_t^T B_i(s, x^*(s)) e^{-r(s-\tau)} ds + g^i[x_T^*] e^{-r(T-\tau)} \,\middle|\, x(t) = x_t^* \right. \right\}$$

$$= \frac{V^{(\tau)i}(t, x_t^*)}{\sum_{j=1}^{n} V^{(\tau)j}(t, x_t^*)} W^{(\tau)}(t, x_t^*) = \frac{V^{(t)i}(t, x_t^*)}{\sum_{j=1}^{n} V^{(t)j}(t, x_t^*)} W^{(t)}(t, x_t^*) \, e^{-r(t-\tau)}$$

$$= \xi^{(t)i}(t, x_t^*) \, e^{-r(t-\tau)}, \qquad \text{for } x_t^* \in X_t^* \text{ and } t \in [\tau, T], \tag{6.2}$$

to denote the expected present value (with initial time set at τ) of nation *i*'s cooperative payoff over the time interval $[t,T]$.

Theorem 5.1. **A** distribution scheme with a terminal payment $-g^i[x_T^* - \bar{x}^i]$ at time T and an instantaneous payment at time $\tau \in [t_0, T]$:

$$B_i\,(\tau, x_\tau^*) = -\left[\xi_t^{(\tau)i}(t, x_t^*)\Big|_{t=\tau}\right] - \frac{1}{2}\sum_{k,j}\sigma^{kj}(x_t^*)\,\xi_{x_k^* x_j^*}^{(t_0)i}(t, x_t^*)\Big|_{t=\tau}$$
$$-\left[\xi_{x_t^*}^{(\tau)i}(t, x_t^*)\Big|_{t=\tau}\right]\left[\sum_{j=1}^{n} a_j\{\hat{q}^j[\psi(\tau, x_\tau^*),\tau], \psi_j(\tau, x_\tau^*)\} - \sum_{j=1}^{n} b_j[\varpi_j(\tau, x_\tau^*), x_\tau^*] - \delta(x_\tau^*)\,x_\tau^*\right],$$
$$\text{for } i \in N, \qquad (6.3)$$

yield Condition 4.1.

Proof. Since $\xi^{(\tau)i}(t, x_t^*)$ is continuously differentiable in t and x_t^*, using (6.2) and Remarks 3.1 and 4.1 one can obtain:

$$E_\tau\left\{ \int_\tau^{\tau+\Delta t} B_i\,(s, x^*(s))e^{-r(s-\tau)}ds \;\middle|\; x(\tau) = x_\tau^* \right\}$$
$$= E_\tau\left\{ \xi^{(\tau)i}(\tau, x_\tau^*) - e^{-r\Delta t}\xi^{(\tau+\Delta t)i}(\tau + \Delta t, x_{\tau+\Delta t}^*) \;\middle|\; x(\tau) = x_\tau^* \right\}$$
$$= E_\tau\left\{ \xi^{(\tau)i}(\tau, x_\tau^*) - \xi^{(\tau)i}(\tau + \Delta t, x_{\tau+\Delta t}^*) \;\middle|\; x(\tau) = x_\tau^* \right\}, \qquad (6.4)$$

for $i \in N$ and $\tau \in [t_0, T]$,

where $\Delta x_\tau = \left[\sum_{j=1}^{n} a_j\{\hat{q}^j[\psi(\tau, x_\tau^*),\tau], \psi_j(\tau, x_\tau^*)\} - \sum_{j=1}^{n} b_j[\varpi_j(\tau, x_\tau^*), x_\tau^*] - \delta(x_\tau^*)\,x_\tau^*\right]\Delta t$

$+\,\sigma(x_\tau^*)\Delta z_\tau + o(\Delta t)$,

$\Delta z_\tau = z(\tau + \Delta t) - z(\tau)$, and $E_\tau[o(\Delta t)]/\Delta t \to 0$ as $\Delta t \to 0$.

With $\Delta t \to 0$, condition (6.4) can be expressed as:

$$E_\tau\left\{ B_i\,(\tau, x_\tau^*)\Delta t + o(\Delta t) \right\} = E_\tau\left\{ -\left[\xi_t^{(\tau)i}(t, x_t^*)\Big|_{t=\tau}\right]\Delta t - \left[\xi_{x_t^*}^{(\tau)i}(t, x_t^*)\Big|_{t=\tau}\right]\right.$$
$$\times\left[\sum_{j=1}^{n} a_j\{\hat{q}^j[\psi(\tau, x_\tau^*),\tau], \psi_j(\tau, x_\tau^*)\} - \sum_{j=1}^{n} b_j[\varpi_j(\tau, x_\tau^*), x_\tau^*] - \delta(x_\tau^*)\,x_\tau^*\right]\Delta t$$

$$-\frac{1}{2}\sum_{k,j}\sigma^{kj}(x_t^*)\,\xi^{(t_0)i}_{x_k^* x_j^*}(t,x_t^*)\Big|_{t=\tau}\Delta t-\left[\xi^{(\tau)i}_{x_t^*}(t,x_t^*)\Big|_{t=\tau}\right]\sigma(x)\Delta z_\tau-o(\Delta t)\ \Bigg\}, \qquad (6.5)$$

Taking expectation and dividing (6.5) throughout by Δt, with $\Delta t \to 0$, yields (6.3). Hence Theorem 5.1 follows. ♦

Finally, explicit illustrative examples of the theoretical framework can be found in Yeung (2007) and Yeung and Petrosyan (2008).

7. Concluding Remarks

A continuation of this study is to apply the above analysis to the WITCH Model proposed by Bosetti et al (2006). To construct a dynamically consistent side payment scheme in the WITCH framework, we assume that in the game players are restricted to use open-loop strategies only (in the spirit of the WITCH model). Therefore well-defined and computable non-cooperative equilibria are readily available in the WITCH framework. If cooperative pollution management policies are agreed upon by the players they will act cooperatively to maximize their joint payoff and design a scheme to share their gains. A cooperative solution and a cooperative state trajectory will be obtained using the WITCH solution scheme. After the cooperative game has been played for some time and if there is any disagreement all the players will revert to play non-cooperatively. The players will consent on an imputation system (like sharing the gains equally or sharing the gains proportional to the size of their non-cooperative payoffs). A discrete approximation of a subgame consistent payment procedure could be obtained under the following proposed scheme:

(1) Within the entire cooperative game duration, payments are made for regular (though can be short) intervals instead of instantaneous payments at every time instant

(actually this is not just for the convenience of our analysis but for real practical reasons).

(2) In any of the payment intervals each player will receive the sum of

(i) his due cooperative payment within that interval, and

(ii) the compensation for the difference in the impacts of the change in the state variables along the cooperative trajectory on his cooperative payoffs and those on his non-cooperative payoff.

Note that only open-loop non-cooperative equilibrium is attainable and if cooperation stops all players will revert to the open-loop non-cooperative mode until the end of the game. Therefore the open-loop non-cooperative equilibria evaluated along the cooperative state trajectory can serve a similar role as that of non-cooperative feedback equilibria in deriving a subgame consistent payment scheme under cooperation.

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